

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2018/2019

DIM5068 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

29 MAY 2019

9.00 a.m. – 11.00 a.m.

(2 Hours)

INSTRUCTIONS TO STUDENT

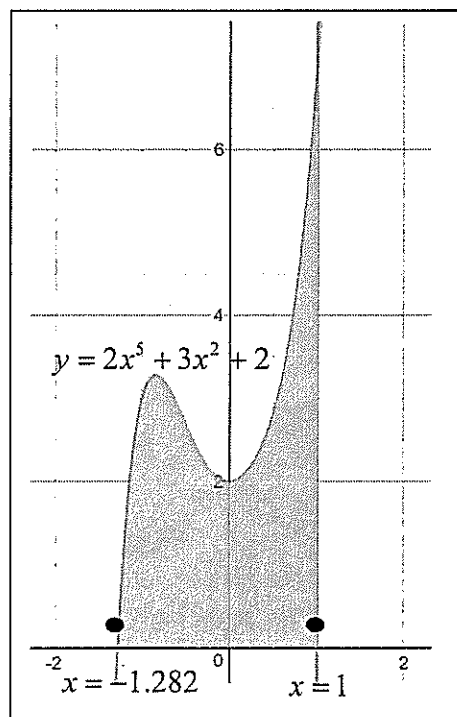
1. This question paper consists of 2 pages with 4 questions excluded the cover page and Appendix. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

- a. Differentiate the function, $y = \frac{-x^4}{3x^2 + 9}$ by using Quotient Rule. (4 marks)
- b. Identify the value of w if the first derivative of function, $y = 10(\tan x)^8$ is $\frac{dy}{dx} = w(\tan x)^7 \sec^2 x$. [Hint: By using Chain Rule] (4 marks)
- c. By using implicit differentiation, show that the first derivative of $3x^4 - 6y^2 = 7$ is $\frac{dy}{dx} = \frac{x^3}{y}$. (3 marks)
- d. Given $f(x) = -3x^3 + 4x^2 + 1$, identify the coordinate of inflection point. (4 marks)

[TOTAL 15 MARKS]**Question 2**

- a. Integrate the following indefinite integral. $\int \left(\frac{3}{x} + 10e^{5x} - 2\pi \right) dx$ (2 marks)
- b. Use Substitution Rule to find $\int 3(18x^5 + 7)(3x^6 + 7x - 1)^8 dx$. (4 marks)
- c. Determine $\int 2x(\cos x) dx$ by using the Integration by Parts. (4 marks)
- d. The diagram below shows the curve of $y = 2x^5 + 3x^2 + 2$. Find the area of the shaded region. (5 marks)

**[TOTAL 15 MARKS]****Continued.....**

Question 3

- a. Solve the separable differential equation, $\frac{dy}{dx} = \frac{4x^3 - 1}{y^2 + 12y}$. (2 marks)
- b. To solve the differential equation, $\frac{dy}{dx} + 2y = e^{-2x}$,
- identify $p(x)$ and $q(x)$. (1 mark)
 - determine the integrating factors, μ . (1 mark)
 - show that the solution of the differential equation is $e^{2x}y = x + 1$ if given that the initial condition is $y(0) = 1$. (3 marks)
- c. Find the general solution of homogeneous equation, $y'' + 2y' + 2y = 0$. (3 marks)

[TOTAL 10 MARKS]**Question 4**

Given vectors, $\mathbf{m} = 3\mathbf{i} - 5\mathbf{k}$ and $\mathbf{n} = 2\mathbf{j} + \mathbf{k}$.

- a. Find
- $2\mathbf{m} + 3\mathbf{n}$. (2 marks)
 - magnitude of $2\mathbf{m} + 3\mathbf{n}$. (1 mark)
 - cross product of $\mathbf{m}$ and $\mathbf{n}$. (2 marks)
 - dot product of $\mathbf{m}$ and $\mathbf{n}$. (1 mark)
- b. Find parametric equations and symmetric equation of the line that passes through the point $(-3, 2, 0)$ and is parallel to vector $\mathbf{m}$. (4 marks)

[TOTAL 10 MARKS]**End of Page**

APPENDIX

Derivatives: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Differentiation Rules

General Formulae

$$\begin{aligned} 1. \frac{d}{dx}[f(x)g(x)] &= f(x)g'(x) + g(x)f'(x) & 2. \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] &= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \\ 3. \frac{d}{dx}(x^n) &= nx^{n-1} & 4. \frac{d}{dx}[f(u)] &= \frac{dy}{du} \cdot \frac{du}{dx} \end{aligned}$$

Exponential and Logarithmic Functions

$$\begin{aligned} 1. \frac{d}{dx}(e^x) &= e^x & 2. \frac{d}{dx}(a^x) &= a^x \ln a \\ 3. \frac{d}{dx}(\ln x) &= \frac{1}{x} & 4. \frac{d}{dx}(\log_a x) &= \frac{1}{x \ln a} \end{aligned}$$

Trigonometric Functions

$$\begin{aligned} 1. \frac{d}{dx}(\sin x) &= \cos x & 2. \frac{d}{dx}(\cos x) &= -\sin x \\ 3. \frac{d}{dx}(\tan x) &= \sec^2 x & 4. \frac{d}{dx}(\csc x) &= -\csc x \cot x \\ 5. \frac{d}{dx}(\sec x) &= \sec x \tan x & 6. \frac{d}{dx}(\cot x) &= -\csc^2 x \end{aligned}$$

Table of Integrals

$$\begin{aligned} 1. \int u \, dv &= uv - \int v \, du & 2. \int u^n \, du &= \frac{u^{n+1}}{n+1} + C, \quad n \neq -1 \\ 3. \int \frac{du}{u} &= \ln|u| + C & 4. \int e^u \, du &= e^u + C \\ 5. \int \sin u \, du &= -\cos u + C & 6. \int \cos u \, du &= \sin u + C \\ 7. \int \sec^2 u \, du &= \tan u + C & 8. \int \csc^2 u \, du &= -\cot u + C \\ 9. \int \sec u \tan u \, du &= \sec u + C & 10. \int \csc u \cot u \, du &= -\csc u + C \end{aligned}$$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)] \, dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \Rightarrow \mu y = \int \mu q(x) dx, \text{ where } \mu = e^{\int p(x) dx}$$

Constant Coefficient of Homogeneous Equations

$$\text{Roots of Auxiliary Equation, } r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

General Solutions to the Auxiliary Equation:

$$2 \text{ Real \& Unequal Roots } (b^2 - 4ac > 0) \quad y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

$$\text{Repeated Roots } (b^2 - 4ac = 0) \quad y = c_1 e^{rx} + c_2 x e^{rx}$$

$$2 \text{ Complex Roots } (b^2 - 4ac < 0) \quad y = e^{\alpha x} (c_1 \cos bx + c_2 \sin bx)$$

Constant Coefficient of Non-Homogeneous Equations

$$y = y_c + y_p \quad [y_c : \text{complementary solution, } y_p : \text{particular solution}]$$

Vector

Length of Vector

$$\text{The length of the vector } \mathbf{a} = \langle a_1, a_2, a_3 \rangle \text{ is } |\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta$$

Area for parallelogram PQRS

$$= \left| \vec{PQ} \times \vec{PR} \right|$$

Area for triangle PQR

$$= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

$$\text{Symmetric equation: } \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

$$\text{Angle between Two Planes: } \theta = \cos^{-1} \left(\frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1| |\mathbf{n}_2|} \right)$$